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FINITE ELEMENT COMPUTATION OF THE BEHAVIORAL MODEL OF MAT FOUNDATION

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ABSTRACT

In this work the influence of soil mechanical properties on the displacements of mat foundation is studied. It was introduced the soil-structure interaction that is modeled by two parameters, the modulus of subgrade vertical reaction (***k***) and the modulus of subgrade horizontal reaction (***2T***). These two parameters are dependent on the geometrical and mechanical characteristics of the system. It appears from this study that the modulus of vertical subgrade reaction is not an intrinsic characteristic but depends on the parameters of the soil and the concrete (E_s ν_s, E_b *and* ν_b) and the dimensions of the plate (*so dependent on the superstructure*). It is clear from this analysis that the foundation soil parameters are more influential than those of the plate.

Keywords: Mat foundation, Soil-structure interaction, Mechanical properties, Finite Element Computation.

1. INTRODUCTION

Developments in civil engineering constructions and especially disorders observed in the supporting structures of civil engineering works pushed the designers to better take into account soil-structure interaction in the process of calculating the foundation structures. Thus, several authors have worked on the modulus of subgrade reaction that is an important parameter in consideration of the soil-structure interface.

This work is a contribution to the finite element analysis of foundation slab resting on elastic soil. The behavioral model is established, as well as all elementary matrices of the model and the matrix assembly. For solving the finite element model, the numerical solver FreeFem© will be used.

In the following, after solving the fundamental equation, the influence of different parameters (**E_s ν_s, E_b**, **ν_b** *and* **e**) on the behavioral model of the foundation will be highlighted.

2. PRESENTATION OF THE CALCULATION MODEL

For analysis of raft foundations, the soil is considered as a springs assembly (with elastic modulus **k**) infinitely close to each other and connected by an elastic membrane (horizontal modulus of reaction = ***2T***) (***Figure 1***).

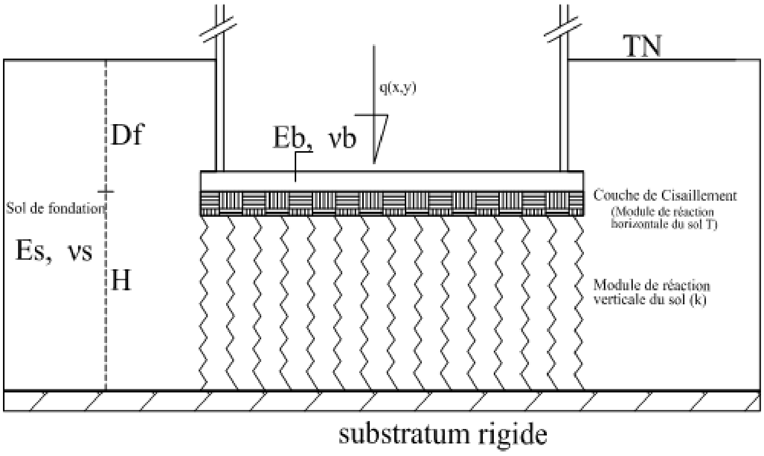

Figure 1 A schematisation of the problem

The theory of plate accounting for the soil-structure interaction *(biparametric model)* leads to mat foundation behavioral law (equation 1):

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)-2T\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+kw=q(x,y) \quad (1)$$

Where **D** is the flexural rigidity of the plate and is given by:

$$D=\frac{E_b e^3}{12(1-\nu_b{}^2)} \quad (2)$$

With:
E_b : elastic modulus of the material constituting the plate,
e: the thickness of the plate;
ν_b : Poisson's ratio of the plate;
k : is the modulus of subgrade reaction.
w : is the deflection

The parameter **k** has been studied by several authors. All authors coming after Biot (1935) tend to give higher values to the soil reaction modulus based on input parameters (Sall, 2015). As the goal is to better understand the deformations, in this research (*for more security*), it would be better to use in the calculations the reaction modulus equation proposed by Biot (1935). In this equation, the displacements of plate points increase if the soil modulus of reaction increases. This equation is expressed as follow:

$$k = \frac{0{,}65E_S}{1-\nu^2}\sqrt[12]{\frac{E_S B^4}{EI}} \qquad (3)$$

Equation 3 has been improved by Vesic (1963), as follow:

$$k = \frac{0{,}95E_S}{1-\nu^2}\left(\frac{E_S B^4}{(1-\nu^2)E_b I}\right)^{0{,}108} \qquad (4)$$

where:
E_s is the modulus of subgrade,
ν_s is the Poisson's ratio of the subgrade;
B is the width of the foundation;
E_b is the Young modulus of the concrete foundation;
I is the moment of inertia of the cross section of the concrete.
T is the horizontal elastic modulus of subgrade reaction. Vlasov (1949) proposes the following relation:

$$T = \frac{E_S}{4(1-\nu_S{}^2)(1+\nu_S(1-\nu_S))}\int_0^H \Phi^2\, dz \qquad (5)$$

To a relatively deep layer of soil where the normal stress may vary with depth, it is possible to use, for the function *Φ* (z), the non-linear continuous variable defined by equation ***5a***. ***Φ(z)*** is a function which describes the variation of the displacement *w(x, y)* along the z axis, such that:

$$\Phi(0) = 1 \; and \; \Phi(H) = 0$$

Selvadurai (1979) suggests two expressions of *Φ (z)*, respectively:

$$\Phi(z) = \left(1 - \frac{z}{H}\right) \qquad (6a)$$

$$\Phi(Z) = \frac{\sinh\left[(H-z)\frac{\gamma}{L}\right]}{\sinh\left(\frac{\gamma H}{L}\right)} \qquad (6b)$$

H: thickness of the soil layer (depth of the rigid substratum).

3. STUDY OF THE PARAMETER K

Figures 2-5 present the evolution of the modulus of subgrade reaction as a function of various parameters of the behavioral model. These figures show a decrease in the subgrade modulus reaction, highly dependent on soil parameters (***E_s***, **ν_s**) and the geometry of the structure. For some fixed parameters, the modulus of subgrade reaction varies very slightly with the mechanical properties of concrete foundation which suggests that changes in the displacements of the structure will be more dependent on mechanical parameters of the subgrade that those of the concrete foundation . Figure 2 shows an increase in the subgrade reaction modulus with the values of the foundation width. For a fixed value of ***B*** (***Figure 3***), the modulus of subgrade reaction is strongly influenced by soil parameters (***E_s***, ***ν_s***). These figures also show that for some fixed parameters, the modulus of subgrade reaction varies very slightly with the mechanical properties of concrete foundation (***Figure 4***), so the changes in displacements of the structure will be more related to subgrade mechanical parameters than to those of the concrete foundation.

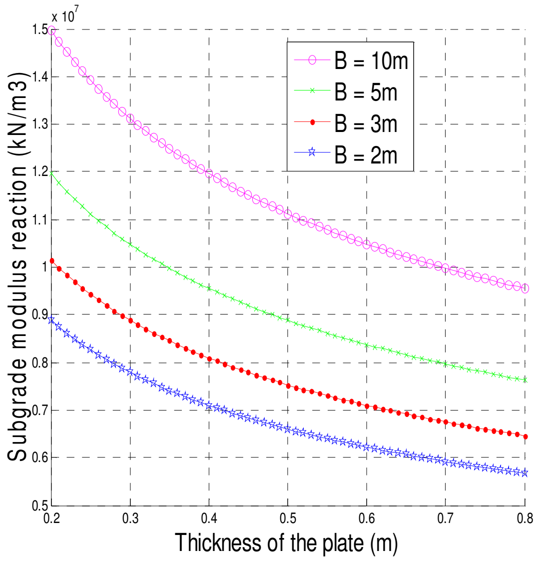

Figure 2 - Modulus of subgrade reaction according to the plate thickness for various values of width of the plate

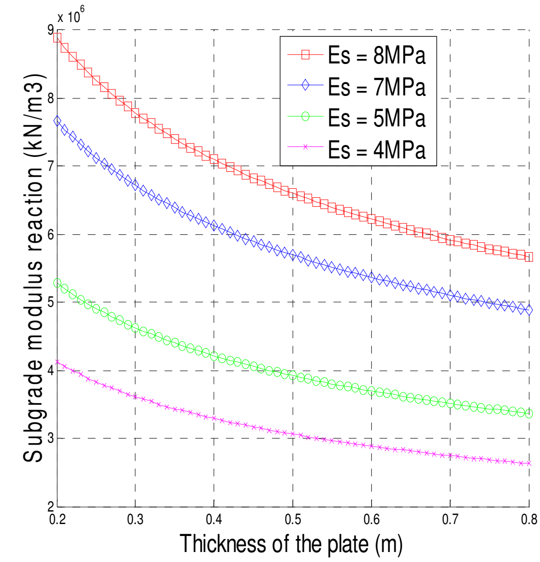

Figure 3 - Modulus of subgrade reaction according to the plate thickness for various values of elastic modulus of the subgrade

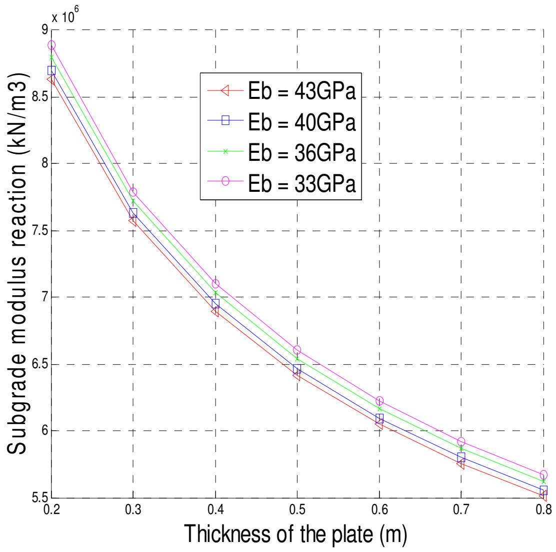

Figure 4 - Modulus of subgrade reaction according to the plate thickness for various values of elastic modulus of concrete foundation

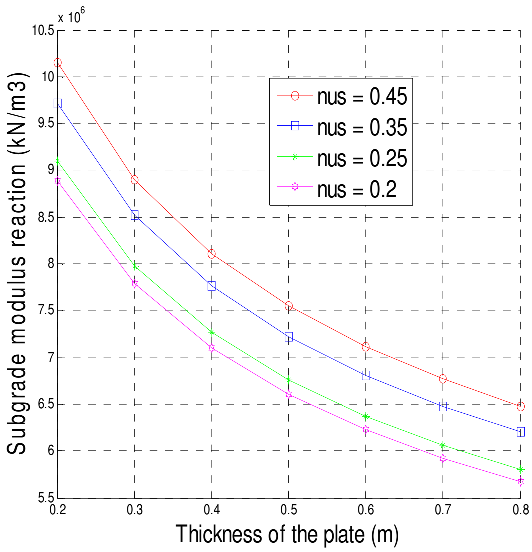

Figure 5 - Modulus of subgrade reaction according to the plate thickness for various values of $\mathbf{\nu_s}$

4. - CALCULATION USING THE FINITE ELEMENT METHOD

4.1 - Development of the stiffness matrix for a plate element

The type of plate element used in this research is rectangular with dimensions ***2a*** and ***2b*** in the ***x*** and ***y*** directions, respectively, and a thickness ***e*** (***Figure 6***). This element is called MZC element because it was originally developed by Melosh (1963), Zienkiewicz and Cheung (1964) as shown in **Figure 6**. The nodal displacements corresponding to each node are:

$$w_i, \frac{\partial w_i}{\partial y} \text{ and } \frac{\partial w_i}{\partial x} \text{ (i= 1, 2, 3,4)} \tag{7}$$

The function of displacement chosen for this element is:

$$w = [N]\{w_e\} \tag{8}$$

where:
$\{w_e\}$ is the nodal displacement vector containing 12 components.
The matrix [N] containing functions of displacement (MZC rectangle shaped), is given by:

$$[N] = [N_1 N_2 N_3 N_4] \tag{9}$$

where for each node ***i***:

$$[N_i] = [N_{i1}, N_{i2}, N_{i3}] \text{ for i= 1,2,3,4} \tag{10}$$

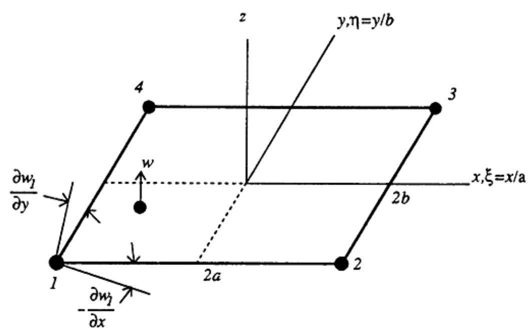

Figure 6 - Rectangle finite element MZC

According to Weaver and Johnson (1984), the shape functions are:

$$N_{i1} = \frac{1}{8}(1+\xi_0)(1+\eta_0)(2+\xi_0+\eta_0-\xi^2-\eta^2) \tag{11}$$

$$N_{i2} = -\frac{1}{8}b\eta_i(1+\xi_0)(1-\eta_0)(1+\eta_0)^2 \tag{12}$$

$$N_{i3} = \frac{1}{8}a\xi_i(1-\xi_0)(1+\eta_0)(1+\xi_0)^2 \tag{13}$$

and

$$\xi_0 = \xi_i\xi \text{ , } \eta_0 = \eta_i\eta \text{ (i= 1, 2, 3,4)} \tag{14}$$

where the ξ_i and η_i are given by the following **table 1**:

Table 1 - Nodal coordinates for a finite element rectangle MZC

i	1	2	3	4
$\boldsymbol{\xi_i}$	-1	+1	+1	-1
$\boldsymbol{\eta_i}$	-1	-1	+1	+1

The linear differential operator generalized {d} is defined such that:

$$(\bar{d}) = -\left\{\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, 2\frac{\partial^2}{\partial x \partial y}\right\}, \tag{15}$$

and

$$\{\Phi\} = \{\bar{d}\}w \tag{16}$$

The generalized strain-displacement matrix can be given by:

$$[B] = [B_1 B_2 B_3 B_4] \tag{17}$$

where

$$[B_i] = \{\bar{d}\}[N_i] = \begin{bmatrix} \frac{\partial^2 N_{i1}}{\partial x^2} & \frac{\partial^2 N_{i2}}{\partial x^2} & \frac{\partial^2 N_{i3}}{\partial x^2} \\ \frac{\partial^2 N_{i1}}{\partial y^2} & \frac{\partial^2 N_{i2}}{\partial y^2} & \frac{\partial^2 N_{i3}}{\partial y^2} \\ 2\frac{\partial^2 N_{i1}}{\partial x \partial y} & 2\frac{\partial^2 N_{i2}}{\partial x \partial y} & 2\frac{\partial^2 N_{i3}}{\partial x \partial y} \end{bmatrix} \qquad (i= 1, 2, 3, 4) \tag{18}$$

From the generalized curvatures, generalized moments can be calculated by:

$$\{M\} = [D]\{\Phi\} = [D][B]\{w_e\} \tag{19}$$

The stiffness matrix of the plate element $[K_p^e]$ is given by:

$$[K_p^e] = \int_{\Omega e} [B]^T\, [D][B] dA = ab \int_{-1}^{1} \int_{-1}^{1} [B]^T [D][B]\, d\xi d\eta \tag{20}$$

After these matrix multiplications and integrations, the stiffness matrix can be obtained for a finite element of plate (Zienkiewicz, 1977):

$$[K_p^e] = \frac{Ee^3}{12(1-\nu^2)} [k_1 + k_2 + k_3 + k_4] \tag{21}$$

where matrix $\boldsymbol{k_1}$, $\boldsymbol{k_2}$, $\boldsymbol{k_3}$ and $\boldsymbol{k_4}$ can be calculated.

4.2-Calculation of equivalent nodal forces

For a plate with a distributed load ***(q)***, the equivalent nodal loads are calculated with the following equations (*Weaver and Johnston, 1984*):

$$f_i = \int_\Omega N_i^T q dA \tag{22}$$

Where

$$f_i = ab \int_{-1}^{1}\int_{-1}^{1} N_i^T q d\xi d\eta \tag{23}$$

After development, the load vector for an element of plate can be obtained by:

4.3 - Determination of matrix modeling soil-structure interaction

It is shown that the stiffness matrix ***[K$_e$]*** and the vector load ***{f$_e$}*** of a finite element of a Kirchhoff plate can be obtained by deriving the potential energy of the internal and external forces acting on the plate. The rigidity of the sub-floor should be extracted from the soil deformation energy. For clarity, the total strain energy in the plate element and the subgrade are expressed from the work of Turhan (1992):

$$U_e = \frac{1}{2}\int_{\Omega_e}\left[\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x \partial y}\right]^T D\left[\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, 2\frac{\partial^2 w}{\partial x \partial y}\right] dA + \frac{1}{2}\int_{\Omega_e}[w(x,y)]^T k[w(x,y)]dA + \frac{1}{2}\int_{\Omega_e}\left[\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right]^T 2t\left[\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right] dA \tag{24}$$

where/
Ω_e is the area of a plate element, and all other terms have been previously defined.

The first part of the above equation shows the conventional stiffness matrix of the plate$[k_P^e]$, differentiation of the second integral over settings (*nodal displacements*) returns a matrix$[k_k^e]$, which represents the effect of axial stress in the soil. And the last term gives a matrix $[k_t^e]$ which represents the effect of shearing in the soil. Thus, the total strain energy of a plate element can be written in the form:

$$U_e = \frac{1}{2}\{w_e\}^T([k_P^e] + [k_k^e] + [k_t^e])\{w_e\} \tag{25}$$

Thus, the stiffness matrix for an element of the plate-soil foundation system is:

$$[k^e] = [k_P^e] + [k_k^e] + [k_t^e] \tag{26}$$

4.3.1-Vertical bending stiffness matrix$[\boldsymbol{k_k^e}]$

The total strain energy in the soil in the vertical direction is:

$$(U_k)_e = \frac{1}{2}\int_{\Omega_e}[w(x,y)]^T k[w(x,y)]dA \tag{27}$$

For each column of soil under the plate element, the first stiffness matrix,$[\boldsymbol{k_k^e}]$, is calculated by minimizing the total energy ***$(U_k)_e$*** by a ratio to each component of the displacement vector $\{\boldsymbol{w_i}\}$:

$$[k_k^e]_{ij} = \frac{\partial^2 (U_k)_e}{\partial w_i \partial w_j} \tag{28}$$

Using the dimensionless coordinates (***ξ, η***) and combining (27) and (28) yields:

$$[k_k^e]_{ij} = \frac{1}{2} kab \frac{\partial^2}{\partial w_i \partial w_j} \int_{-1}^{1} \int_{-1}^{1} w^2 d\xi d\eta \tag{29}$$

Displacements at any point coordinates (***ξ, η)*** of the plate are constituted by the same form of functions used in assessing the stiffness matrix of a plate element. Referring to the above equations and substituting the shape functions of the plate member in the expression of *w*, give the following stiffness matrix (***12×12)*** to take account of the axial stress on the soil:

$$[k_k^e] = kab \int_{-1}^{1} \int_{-1}^{1} [N]^T [N] d\xi d\eta \tag{30}$$

This matrix was developed by Chilton and Wekezer (1990), but all coefficients are false. Using their coefficients, the finite element model does not provide movement when the rigid plate is uniformly charged. Recognizing this problem, the authors have reassessed all the coefficients of the matrix $[\boldsymbol{k_k^e}]$, and the corrected coefficients gives exactly constant ***q/k*** on the move during the simulation of a Winkler (1932) model for a distributed uniformly charged plate. The matrix is partitioned into four square matrices (6×6):

$$[k_k^e] = kab \begin{bmatrix} k_{k1} & k_{k2} \\ sym. & k_{k3} \end{bmatrix} \tag{31}$$

4.3.2 - Stiffness matrix due to shear deformations$\boldsymbol{k_t^e}$

The total strain energy in the shear effect is:

$$(U_t)_e = \frac{1}{2} \int_{\Omega_e} \left[\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right]^T 2t \left[\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right] dA \tag{32}$$

By minimizing this function of the deformation energy in relation to all the components of the displacement vector of the plate, there are given the second matrix rigidity of the foundation which is expressed as:

$$[k_t^e]_{ij} = \frac{\partial^2 (U_t)_e}{\partial w_i \partial w_j} \tag{33}$$

Combining equations (31) and (32) and using the natural coordinates, it can be obtained the following equation:

$$[k_t^e]_{ij} = \frac{1}{2} (2t) ab \frac{\partial^2}{\partial w_i \partial w_j} \int_{-1}^{1} \int_{-1}^{1} (\nabla w)^2 d\xi d\eta \tag{34}$$

where the displacement function, ***w***, is as defined above.
Substituting the expression of ***w*** in equation (34) and performing the integrations and differentiations, give the second matrix rigidity of the foundation, which is still a 12×12 matrix:

$$[k_t^e]_{ij} = 2tab \int_{-1}^{1} \int_{-1}^{1} \left(\frac{1}{a^2} \left[\frac{\partial N}{\partial \xi}\right]^T \left[\frac{\partial N}{\partial \xi}\right] + \frac{1}{b^2} \left[\frac{\partial N}{\partial \eta}\right]^T \left[\frac{\partial N}{\partial \eta}\right] \right) d\xi d\eta \tag{35}$$

$$[k_t^e] = 2tab \begin{bmatrix} k_{t1} & k_{t2} \\ sym. & k_{t3} \end{bmatrix} \tag{36}$$

5. - DETERMINING OF THE COEFFICIENTS OF THE OVERALL MATRIX OF THE SYSTEM

Using the standard procedure in the method of finite elements for the assembly of the elements, the global stiffness matrix is represented in the form of a half band matrix. The global stiffness matrix for the whole system is symbolically represented by capital letters, such as:

$$[K] = \sum_{e=1}^{N_e}\left(\left[k_p\right] + [k_k] + [k_t]\right) \tag{37}$$

$$[K] = \left[K_p\right] + [K_k] + [K_t] \tag{38}$$

where $\mathbf{N_e}$ is the total number of finite elements of plate.
The final equation to solve is given by:

$$[K]\{W\} \ = \{F\} \tag{39}$$

where ***[K]*** is the global stiffness matrix of the system, ***{W}*** the displacement vector and ***{F}*** is the vector load applied to the system. For solving the finite element model, the numerical solver FreeFem ++ is used.

6. PRESENTATION AND DISCUSSION OF RESULTS

For the study using the finite element method, it is considering a ***10m×10m*** plate subject to a uniformly distributed load. The mechanical properties of soil and concrete foundation vary as described in the analysis by the analytical method. **Figures 7-15** give the following results of the analysis by the finite element method for fixed parameters and zero displacements at the edges.

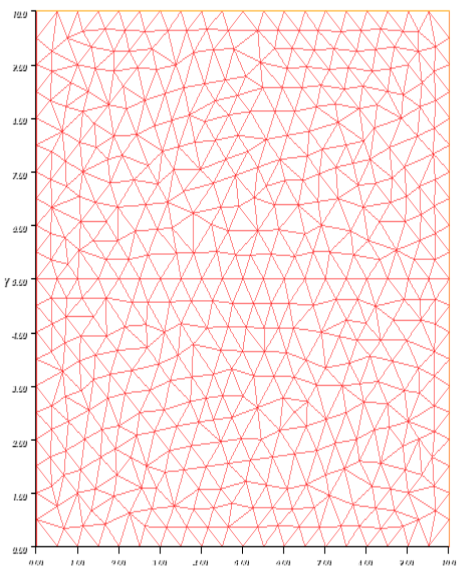

Figure 7 - Mesh of the domain by finite element

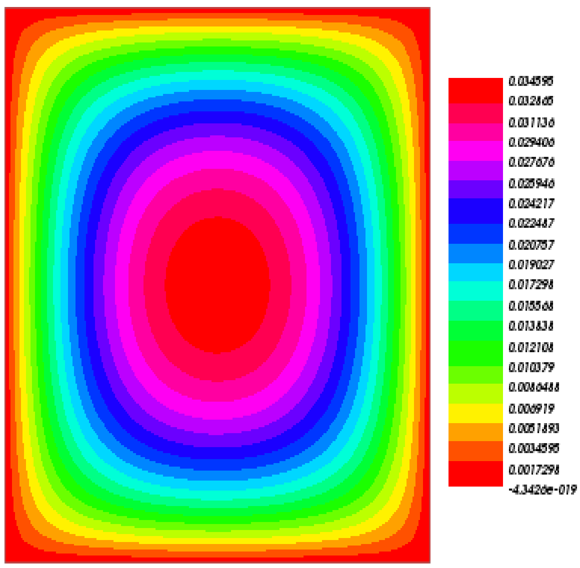

Figure 8 - Viewing of displacements

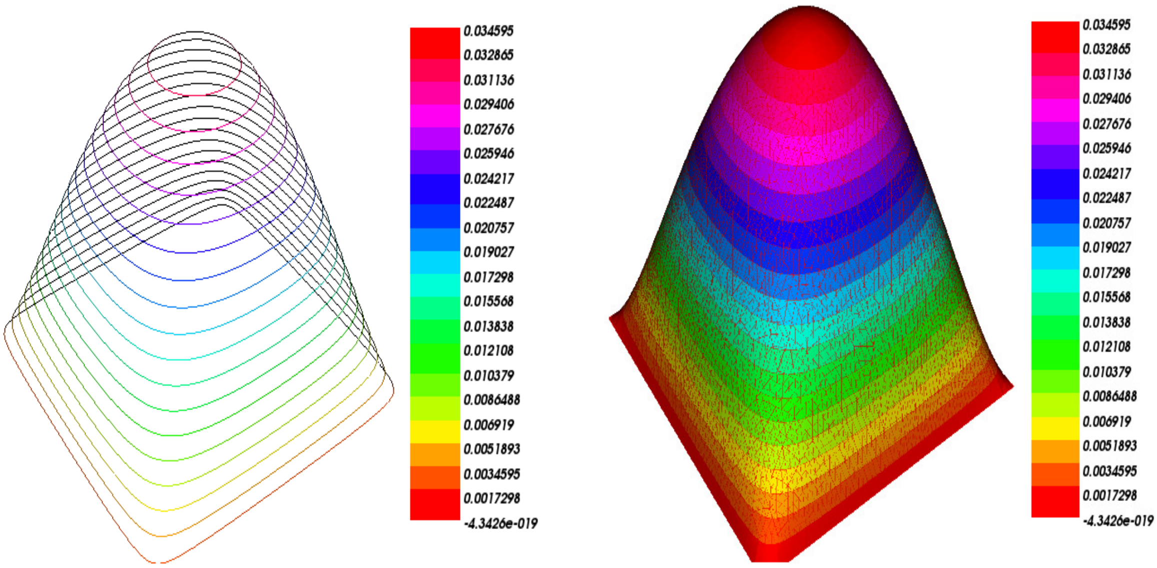

Figure 9 - Isovalues displacements and deformations in 3D with mesh

The following figures show the influence of the model parameters on the displacements of the mat foundation. It appears from the analysis by the finite elements method that the parameters of the soil (E_s, ν_s) have a significant influence on the behavior of the system (**Figures 10** and 12). The elastic modulus of the concrete (E_b) has a very slight influence on the calculation model (**Figure 11**). The behavioral model of the system is almost insensitive to the Poisson ratio (ν_b) of the concrete (**Figure 13**). **Table 2** and **Figure 15** show the sensitivity of the calculation by finite elements. The results of the analysis by the finite elements method are comforting conclusions of analytical calculation. The finite elements reveal almost no lift at the edges of the plate.

Table 2. Sensitivity of finite elements calculation

NEF	*514*	1194	*2128*	*3848*	*5946*
w_{max} *(m)*	*0.0345691*	*0.0345881*	*0.0345966*	*0.0345966*	*0.0345966*

NEF: Number of finite elements; w_{max}*: displacement of the center of the plate.*

Figure 16 gives the results obtained from the investigation of the influence of the ratio ***L/B*** on the movements of the points of the plate. These results show that the higher the ratio ***L/B***, the greater w_{max} is important. However, w_{max} tends to a limited value for large values of ***L/B***.

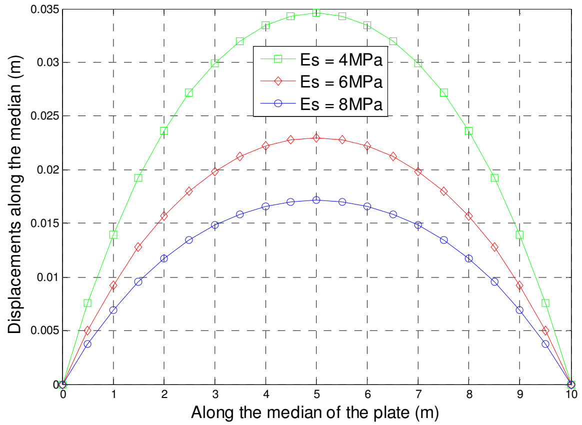

Figure 10 - Displacement profile following the median of the plate for different values of **Es**

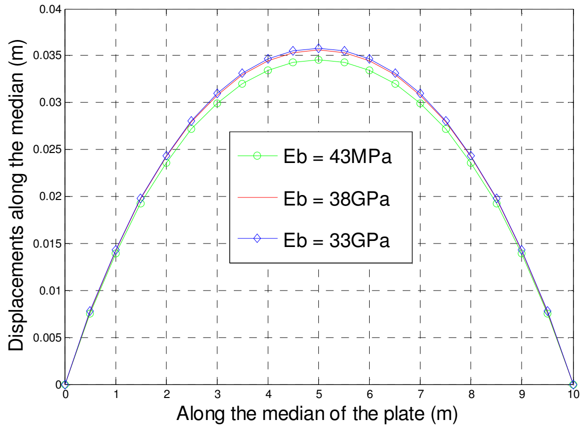

Figure 11 - Displacement profile following the median of the plate for different values of $\mathbf{E_b}$

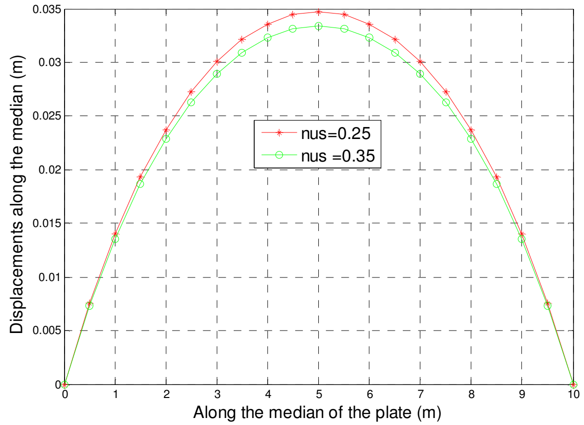

Figure 12 -Displacement profile following the median of the plate for different values of ν_s

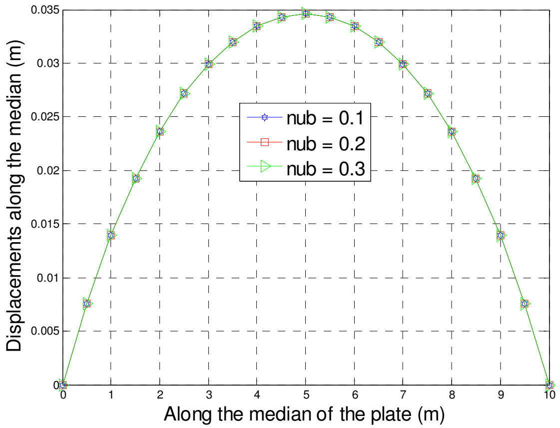

Figure 13- Displacement profile following the median of the plate for different values of $\boldsymbol{\nu_b}$

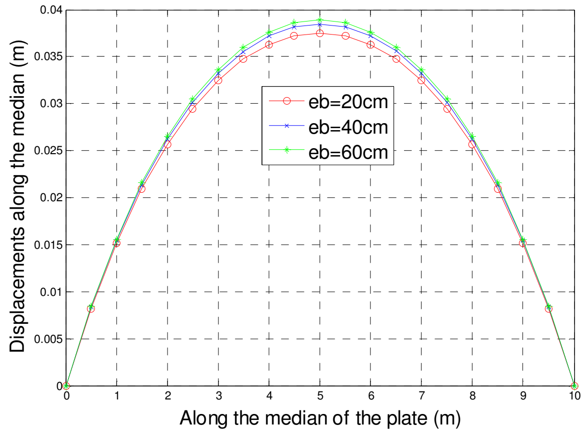

Figure 14- Displacement profile following the median of the plate for different values of $\mathbf{e_b}$

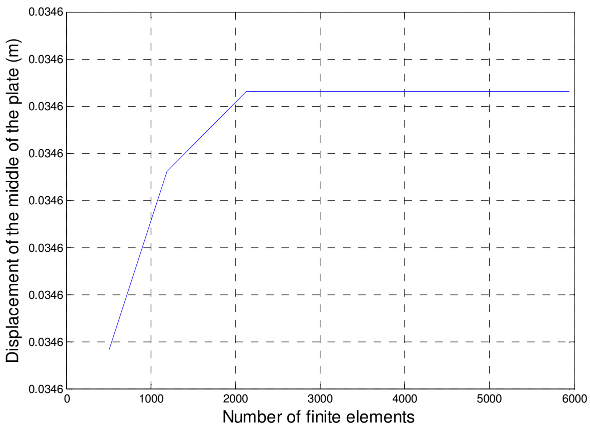

Fig. 15 - Displacements of the center of the plate for various values of the number of finite elements

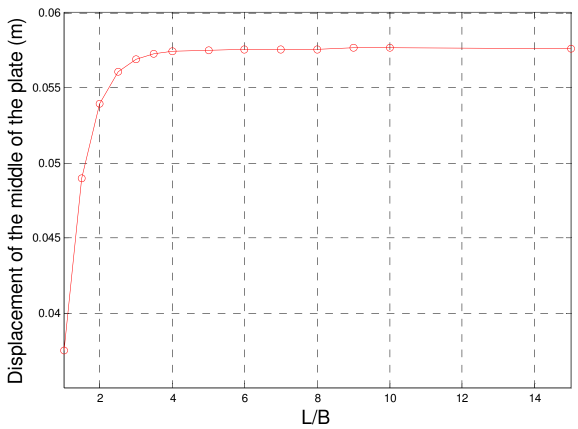

Figure. 16- w_{max} for various value of **L/B**

7. CONCLUSION

After studying the influence settings on the behavioral model of the structure, the use of results leads to strong conclusions and remarks. Following this analysis, the elastic modulus of the subgrade has a very significant influence on the displacements of the concrete and the parameters of the concrete have almost no influence on the movement of the plate. The study also reveals firstly that knowledge of the mechanical properties of soil is a prerequisite for the mastery of the behavior of foundation structures. Also, the development of the foundation soil behavior model must necessarily go through a complete geotechnical characterization of materials. All results (*numerical*) regarding displacements attest that the deformations are not only function of the loads but also function of the mechanical properties of concrete and subgrade, which logically leads to the need to favor a complete geotechnical characterization of materials.

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